MIZORAM PUBLIC SERVICE COMMISSION

GENERAL COMPETITIVE EXAMINATIONS FOR RECRUITMENT TO THE POST OF JUNIOR GRADE OF MIZORAM FOREST SERVICE i.e. ASSISTANT CONSERVATION OF FOREST (ACF) UNDER ENVIRONMENT, FOREST & CLIMATE CHANGE DEPARTMENT, GOVERNMENT OF MIZORAM, 2018

MATHEMATICS

Time Allowed : 3 hours | Full Marks : 100

The figures in the margin indicate full marks for the questions.

Answer any 10 (ten) questions taking 5 (five) questions from each section.

SECTION - A

1. (a) Verify Cayley-Hamilton theorem for the matrix $A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$ and hence find A^{-1}. **(5)**

(b) Determine whether or not the following vectors form a basis of $\mathbb{R}^3$, **(5)**
(1,1,2), (1,2,5) and (5,3,4).

2. (a) Compute A^{-2} where $A = \begin{pmatrix} 1 & 0 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$. **(5)**

(b) Using the elementary row operations, find the rank of the matrix

$$\begin{bmatrix} 3 & -2 & 0 & -1 \\ 0 & 2 & 2 & 1 \\ 1 & -2 & -3 & -2 \\ 0 & 1 & 2 & 1 \end{bmatrix}$$ **(5)**

3. (a) Prove that $B(l,m) = \dfrac{\Gamma(l)\Gamma(m)}{\Gamma(l+m)}$. **(5)**

(b) Let $f : R^2 \to R$ be defined by **(5)**

$$f(x,y) = \frac{xy}{\sqrt{x^2+y^2}},\ (x,y) \neq 0 \text{ and } f(0,0) = 0.$$

Prove that f_x and f_y exist at (0,0). Also prove that f is continuous at (0,0).

4. (a) Show that the plane $x+2y-z=4$ cut the sphere $x^2+y^2+z^2-x+z-2=0$ in a circle of radius unity and find the equation of sphere which has this circle for one of its great circle. **(5)**

(b) Find the equation of the right circular cylinder of radius 2 whose axis passes through (1,2,3) and has direction ratios (2,-3,6). **(5)**

5. (a) Using the method of variation of parameters, solve the differential equation

$$\frac{d^2y}{dx^2}+y= cosex\ x.$$ **(5)**

(b) Solve $x^2\frac{d^2y}{d^2x}$-2x$\frac{dy}{dx}+2y=x^3$. **(5)**

6. (a) A particle projected with a velocity u, strikes at right angles a plane through the point of projection inclined at an angle β to the horizon. Show that the time of flight is $\frac{2u}{g\sqrt{1+3\sin^2\beta}}$. Find also the height of the point struck above the point of projection. **(5)**

(b) A particle moves with S.H.M. in a straight line. In the first second after starting from rest, it travels a distance a and in the next second it travels a distance b in the same direction. Prove that the amplitude of the motion is $\frac{2a^2}{3a-b}$. **(5)**

7. (a) Prove the identity: **(5)**

$$\nabla(F.G)=F\times(\nabla\times G)+G\times(\nabla\times F)+(F.\nabla)G+(G.\nabla)F$$

(b) Evaluate $\iint\left(y^2z^2\hat{i}+z^2x^2\hat{j}+z^2y^2\hat{k}\right)\cdot\hat{n}\,dS,$ where S is the part of the sphere $x^2+y^2+z^2=1$, above the xy-plane and bounded by this plane. **(5)**

SECTION - B

8. (a) Let S be the set of all real numbers except -1. Define * on S by $a*b=a+b+ab$. Is $(S,*)$ a group? Find the solution of the equation $3*x*2=8$ in S. **(5)**

(b) Prove that $(ab)^2=a^2b^2 \Leftrightarrow$ G is abelian where a, $b\in G$. **(5)**

9. (a) Evaluate $\iint(x+y)^2\,dx\,dy$ over the area bounded by the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$. **(5)**

(b) What kind of singularities have the following functions: **(5)**

(i) $\frac{\cot\pi z}{(z-a)^2}\ at\ z=0,\ z=\infty$

(ii) $\left(\frac{z-2}{z^2}\right)\sin\left(\frac{1}{z-1}\right)\ at\ z=0,\ z=1.$

10. (a) Solve by Simplex Method : Maximize $z = 3x_1 + 2x_2$, **(5)**

Subject to the constraints: $x_1 + x_2 \le 4$, $x_1 - x_2 \le 2$, and $x_1, x_2 \ge 0$.

(b) Using the dual solve the following L.P.P. **(5)**

Max. $z_p = 3x_1 - 2x_2$,

Such that

$$x_1 \le 4,$$
$$x_2 \le 6,$$
$$x_1 + x_2 \le 5,$$
$$-x_2 \le -1,$$

and $x_1, x_2 \ge 0.$

11. (a) Find the complete integral of the equation **(5)**

$$2xz - px^2 - 2qxy + pq = 0$$

where $p = \frac{\partial z}{\partial x}$ and $q = \frac{\partial z}{\partial y}$

(b) Solve by Charpit's method : $(p + q)(px + qy) = 1$, $p = \frac{\partial z}{\partial x}$ and $q = \frac{\partial z}{\partial y}$. **(5)**

12. (a) Find the polynomial f(x) by using Lagrange's interpolation formula and hence find f(3) for

x:	0	1	2	5
f(x):	2	3	12	147.

(5)

(b) Solve the differential equation $\frac{d^2y}{dx^2} - 7\frac{dy}{dx} - 6y = (x+1)e^{2x}$ **(5)**

13. (a) Find a positive root of $x^3 - 2x - 5 = 0$, correct to 4 decimals, by Newton-Raphson method. **(5)**

(b) Evaluate $\int_0^6 \frac{dx}{1+x^2}$ by using Trapezoidal rule. **(5)**

14. (a) Derive the Lagrange's equations of motion for a holonomic dynamical system with *n* degrees of freedom. **(5)**

(b) Show that the moment of inertia of ellipse area of mass M and semi- axis *a* and *b* about a diameter of length *r* is $\frac{1}{4}M\frac{a^2b^2}{r^2}$. **(5)**

* * * * * * *