MIZORAM PUBLIC SERVICE COMMISSION

MIZORAM CIVIL SERVICES (COMBINED COMPETITIVE) MAIN EXAMINATION, 2023

MATHEMATICS PAPER-I

Time Allowed : 3 hours FM : 100

Marks for each question is indicated against it.

Attempt any 5 (five) questions taking not more than 3 (three) questions from each Part.

PART - A

1. (a) Show that a nonempty subset S of a vector space V over F is a subspace of V if and only if $a\alpha + b\beta \in S$ whenever a, b ∈ F and a, b ∈ S. **(6)**

(b) Let W be subspace of a finite dimentional vector space V. Then, prove that

$$\dim \frac{V}{W} = \dim V - \dim W.$$ **(8)**

(c) Obtain the characteristic equation of the matrix $A = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix}$ and verify Cayley-Hamilton Theorem. **(6)**

2. (a) Evaluate $\lim_{x \to \infty} \left(1 + \frac{1}{x}\right)^x$. **(4)**

(b) Find all the asymptotes of $x^3 + x^2 y - xy^2 - y^3 - 3x - y - 1 = 0$. **(6)**

(c) If $x = \tan^{-1} \frac{x^3 + y^3}{x - y}$, show that **(3+3=6)**

(i) $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = sin\, 2u$

(ii) $x^2 \frac{\partial^2 u}{\partial x^2} + 2xy \frac{\partial^2 u}{\partial x \partial y} + y^2 \frac{\partial^2 u}{\partial y^2} = \left(1 - 4sin^2 u\right) sin\, 2u$

(d) Show that the maximum rectangle with a given perimeter is a square. **(4)**

3. (a) Evaluate any two of the following : **(2×3=6)**

(i) $\int \frac{2x+1}{x^3 + x^2 - 2x} dx$

(ii) $\int \frac{dx}{a^2 \cos^2 x + b^2 \sin^2 x}$

(iii) $\int \frac{xe^x}{(x+1)^2} dx$

(b) Examine the convergence of improper integral $\int_0^\infty \frac{dx}{(x+1)(x+2)}$ **(4)**

(c) Evaluate $\int_1^e \int_0^{\log y} \int_1^{e^x} \log z \, dy \, dx \, dz$. **(5)**

(d) Find the area of the segment cut off from the parabola $y^2 = 4x$ by the straight line $y = 8x - 1$. **(5)**

4. (a) Reduce the equation $6y^2 - 18yz - 6xz + 2xy - 9x + 5y - 5z + 2 = 0$ to its canonical form. **(8)**

(b) Find the shortest distance between the lines **(6)**

$$\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4};$$

$$\frac{x-2}{3} = \frac{y-3}{4} = \frac{z-4}{5}$$

(c) Find the equation of the cone whose vertex is (1, 2, 3) and guiding curve is the circle **(6)**

$x^2 + y^2 + z^2 = 4, \; x + y + z = 1$

PART - B

5. (a) Find the orthogonal trajectories of family of curves $y = ax^2$, a being a parameter. **(4)**

(b) Solve $y + px = p^2x^4$ **(5)**

(c) Solve : $(1+x)^2 \frac{d^2y}{dx^2} + (1+x)\frac{dy}{dx} + y = 4\cos \log(1+x)$. **(6)**

(d) Show that $L\left\{\frac{\sin t}{t}\right\} = \tan^{-1}\frac{1}{p}$ and hence find $L\left\{\frac{\sin at}{t}\right\}$. Does the Laplace transform of $\frac{\cos at}{t}$ exists? **(5)**

6. (a) A particle of mass m is moving along the axis of x under a central force μmx to the origin. When $t = 2$ seconds, it passes through the origin and when $t = 4$ seconds, its velocity if 4 cm per second. Determine the motion and show that, if the complete period is 16 seconds, the amplitude of the path is $\frac{32\sqrt{2}}{\pi}$ cm. **(6)**

(b) A particle moving in a straight line is acted on by a force which works at a constant rate and changes its velocity from u to v in passing over a distance x. Show that the time taken is

$$\frac{3(u+v)x}{2(u^2+uv+v^2)}$$ **(6)**

(c) A gun of mass M fires a shell of mass m horizontally and the energy of explosion is such as would be sufficient to project the shell vertically to a height h. Show that the velocity of recoil of the gun is $\left\{\frac{2m^2gh}{M(m+M)}\right\}^{\frac{1}{2}}$. **(8)**

7. (a) Forces P,Q,R act along the sides of the triangle formed by the lines $x + y = 1, y - x = 1, y = 2$, find the magnitude and the line of action of the resultant. **(7)**

(b) The least force which will move a weight up an inclined plane is P. Show that the least force, acting parallel to the plane which will move the weight upwards is $P\sqrt{1+\mu^2}$, μ being the coefficient of friction of the plane. **(7)**

(c) A square lamina rests with its plane perpendicular to a smooth wall one corner being attached to a point in the wall by a fine string of length equal to the side of the square. Find the position of equilibrium and show that it is stable. **(6)**

8. (a) Find the value of a for the vector $\vec{A} = (x+3y)\hat{i} + (y-2z)\hat{j} + (x+az)\hat{k}$ is solenoidal. **(6)**

(b) Find the constants l, m, n so that $\bar{u} = (x+2y+lz)\hat{i} + (mx-3y-z)\hat{j} + (4x+ny+2z)\hat{k}$ is irrotational. **(6)**

(c) Using Gauss' divergence theorem, evaluate $\int\int_S \vec{F}.\hat{n}dS$. **(8)**

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