MIZORAM PUBLIC SERVICE COMMISSION

General Competitive Examinations for Recruitment to the post of Jr. Grade of Mizoram Forest Service (Asst. Conservator of Forests) under Environment, Forest & Climate Change Department, Government of Mizoram, 2023

MATHEMATICS

Time Allowed : 3 hours | Full Marks : 100

The figures in the margin indicate full marks for the questions.

Answer any 10 (ten) questions taking 5 (five) questions from each section.

SECTION - A

1. (a) Find the inverse of the matrix, $A = \begin{bmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{bmatrix}$. **(5)**

(b) For the 3-dimensional space $\mathbb{R}^3$ over the field of real numbers $\mathbb{R}$, determine whether the set {(2,-1,0), (3,5,1), (1,1,2) is a basis. **(5)**

OR

(c) Let the mapping $T : \mathbb{R}^3 \rightarrow \mathbb{R}^3$ defined by

$T(x_1, x_2, x_3) = (x_1 + x_2 + x_3, 2x_1 + x_2 + 2x_3, x_1 + 2x_2 + x_3); (x_1, x_2, x_3) \in \mathbb{R}^3$.

Show that T is linear mapping. **(5)**

(d) Let V be the vector space of all 2×2 matrices over R. Let $W_1 = \left\{ \begin{pmatrix} x & y \\ z & 0 \end{pmatrix} \middle/ x, y, z \in R \right\}$;

$W_2 = \left\{ \begin{pmatrix} x & 0 \\ 0 & z \end{pmatrix} \middle/ x, z \in R \right\}$. Prove that W_1, W_2 are subspaces of V. **(5)**

2. (a) For what values of a, b and c if any, does the function,

$f(x) = \begin{cases} x^2, & x \le 0 \\ ax + b, & 0 < x \le 1 \\ c, & x > 1 \end{cases}$ is differentiable at $x = 0$ and $x = 1$. **(5)**

(b) Find the area bounded by the parabola $x^2 = 8y$ and the line $x - 2y + 8 = 0$. **(5)**

OR

(c) Find the asymptote of the curve $3x^3+2x^2y-7xy^2+2y^3-14xy+7y^2+4x+5y=0$. **(5)**

(d) If $u=\log\sqrt{x^2+y^2+z^2}$, prove that $(x^2+y^2+z^2)\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}+\frac{\partial^2 u}{\partial z^2}\right)=1$. **(5)**

3. (a) Find the equation of the cone whose vertex is the point (α,β,γ) and whose generating lines pass through the conic $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1, z=0$. **(5)**

(b) Find the length and the equations of the shortest distance between the lines.

$\frac{x-1}{2}=\frac{y-2}{3}=\frac{z-3}{4};\frac{x-2}{3}=\frac{y-4}{4}=\frac{z-5}{5}$. **(5)**

OR

(c) For what value of λ does the equation $12x^2-10xy+2y^2+11x-5y+\lambda=0$

represent two straight lines. Show that the angle between them is $\tan^{-1}\left(\frac{1}{7}\right)$. **(5)**

(d) Show that the plane $8x-6y-z=5$ touches the paraboloid $\frac{x^2}{2}-\frac{y^2}{3}=z$. **(5)**

4. (a) By substituting $x^2 = u$ and $y^2 = v$, reduce $x^2(y-px) = yp^2$ into Clairaut's form and find the singular solution. **(5)**

(b) Reduce the equation $\frac{dy}{dx}+\frac{1}{x}\sin 2y=x^3\cos^2 y$ to a linear differential equation and solve it. **(5)**

OR

(c) Solve $\sin px\cos y=\cos px\sin y+p$, where $p=\frac{dy}{dx}$. **(5)**

(d) Solve $(x^2D^2-xD+2)y=x\log x$, where $D\equiv\frac{d}{dx}$. **(5)**

5. (a) A particle of mass *m* is falling under the influence of gravity through a medium whose resistance equals μ times the velocity. If the particle be released from rest, show that the distance fallen through in time is $g\frac{m^2}{\mu^2}\left\{e^{\frac{\mu t}{m}}-1+\frac{\mu t}{m}\right\}$. **(5)**

(b) A particle is moving with SHM and while making an excursion from one position of rest to the other, its distance from the middle point of its path at three consecutive seconds are observed to be x_1, x_2 and x_3. Prove that the time of a complete oscillation is $2\pi\Big/\cos^{-1}\left(\frac{x_1+x_3}{2x_2}\right)$. **(5)**

OR

(c) A particle moves with simple harmonic motion in a straight line. In the first second starting from rest, it travels a distance 'a' and in the next second it travels a distance 'b' in the same direction.

Prove that the amplitude of motion is $\dfrac{2a^2}{3a-b}$. **(5)**

(d) A particle is projected in a direction making an angle α with the horizon. If it passes through the point whose co-ordinate are (x_1, y_1) referred to the perpendicular axes through the point of projection. Show that $\tan\alpha = \dfrac{y_1}{x_1}\left(\dfrac{R}{R-x_1}\right)$ where R is the horizontal range. If it also passes through the point (x_2, y_2), prove that $\tan\alpha = \dfrac{x_2^2 y_1 - x_1^2 y_2}{x_1 x_2 (x_2 - x_1)}$. **(5)**

6. (a) Prove that for any three vectors $\vec{a}, \vec{b}, \vec{c}$, $\vec{a}\times(\vec{b}\times\vec{c}) = (\vec{a}\cdot\vec{c})\vec{b} - (\vec{a}\cdot\vec{b})\vec{c}$. **(5)**

(b) Verify Stoke's theorem for the function $\vec{F} = x^2\hat{i} + xy\hat{j}$ integrated along the rectangle in the xy-plane whose sides are along the lines x=0, y=0; x=a and y=b. **(5)**

OR

(c) Find $(\vec{A}\times\nabla)\times\vec{B}$ at the point of (1,-1,2), if

$\vec{A} = xz^2\hat{i} + 2y\hat{j} - 3xz\hat{k}$ and $\vec{B} = 3xz\hat{i} + 2yz\hat{j} - z^2\hat{k}$ **(5)**

(d) Find the value of the constant d such that the vectors, $2i - j + k$, $i + 2j - 3k$ and $3i + dj + 5k$ are coplanar. **(5)**

SECTION - B

7. (a) Prove that the intersection of two subgroups of a group G is a subgroup of G. **(5)**

(b) If R is the additive group of real numbers and R^+ the multiplicative group of all positive real numbers. Then prove that the mapping $f : R \to R^+$ defined by $f(x) = e^x$ for all $x \in R$ is an isomorphism of R onto R^+. **(5)**

OR

(c) Prove that a ring R is without zero-divisors if and only if for all $a, b, c \in R$, $ab=ac$ and $a \neq 0$ imply $b=c$. **(5)**

(d) Let G is a finite group. Prove that order of any element of G divides order of G. **(5)**

8. (a) Prove that the improper integral $\displaystyle\int_a^b \frac{dx}{(x-a)^n}$ converges if and only if n < 1. **(5)**

(b) Let $f(x) = x$ over [0,1]. By dissecting [0,1] into n-equal parts and show that f is Riemann integrable. **(5)**

OR

(c) Expand $f(z)\frac{1}{(z+1)(z+3)}$ in power series valid in $0<|z+1|<2$. **(5)**

(d) Find the value of the integral $\int_C \frac{2z^3+2}{(z-1)(z^2+9)}dz$, where $C:|z-2|=2$ described in counterclockwise direction. Use Cauchy integral formula. **(5)**

9. (a) Solve by graphical method. Maximise $z=50x+15y$ subject to the constraints:
$5x+y\leq 100, x+y\leq 60, x,y\geq 0$ **(10)**

OR

(b) Write the dual of the following problem: **(10)**

$Min.\ Z=3x_1+x_2$

such that $2x_1+3x_2\geq 2$

$x_1+x_2\leq 1$

and $x_1,x_2\geq 0$

10. (a) Find the complete integral of the partial differential equation $z-px-qy=p^2+q^2$ by using Charpit's method. **(10)**

OR

(b) Solve $zxp+yzq=xy$. **(10)**

11. (a) Find the positive real root of $x\log_{10}x=1.2$ using the bisection method in three iterations. **(5)**

(b) Find the positive solution of $2\sin x=x$ using Newton-Raphson method. **(5)**

OR

(c) Find the value of $\log 2^{\frac{1}{3}}$ from $\int_0^1 \frac{x^2}{1+x^3}dx$ using Simpson's $1/3$ rule with $h=0.25$. **(5)**

(d) Using Euler's method, solve the ordinary differential equation $\frac{dy}{dx}=1+xy$ given that $y(0)=2$ and find $y(0.3)$. **(5)**

12. (a) Derive the Lagrange's Equations of motion for a holonomic dynamical system specified by the n generalized co-ordinate $q_1=(j=1,2,...n)$. **(10)**

OR

(b) From the translational equation of motion of the form $\frac{du}{dt}=X+\frac{1}{\rho}\left(\frac{\partial p_{xx}}{\partial x}+\frac{\partial p_{yx}}{\partial y}+\frac{\partial p_{zx}}{\partial z}\right)$, obtain the Navier-Stokes's Equations of motion for a viscous fluid. **(10)**

* * * * * * *