MIZORAM PUBLIC SERVICE COMMISSION

MIZORAM CIVIL SERVICES (COMBINED COMPETITIVE) MAIN EXAMINATION, 2023

MATHEMATICS PAPER-II

Time Allowed : 3 hours FM : 100

Marks for each question is indicated against it.

Attempt <u>any 5 (five)</u> questions taking not more than 3 (three) questions from each Part.

PART - A

1. (a) Let $f: G \to G'$ be an onto homomorphism with $Ker\ f = K$. For H8 a subgroup of G8, define $H = \{x \in G : f(x) \in H'\}$. Prove the following: **(4+4=8)**

(i) H is a subgroup of G and $K \subseteq H$

(ii) H8 is a normal subgroup of G8 if and only if H is normal in G.

(b) Prove that an ideal M in Z is a maximal ideal if and only if $M = pZ$ where p is a prime. **(7)**

(c) Prove that every group of prime order is cyclic. **(5)**

2. (a) Let $f_n : [a,b] \to R$ be defined by $f_n(x) = \dfrac{nx}{1+n^2x^2}$, where 0 is an interior point of [a,b]. Prove that the sequence of functions f_n is point-wise convergent but not uniformly convergent. **(6)**

(b) Show that $\int_0^1 x^{m-1}(1-x)^{n-1}\,dx$ exists if and only if m and n are both positive. **(8)**

(c) If
$$f(x,y) = \begin{cases} \dfrac{xy(x^2-y^2)}{x^2+y^2} & \text{when } (x,y) \neq (0,0) \\ 0 & \text{when } (x,y) = (0,0) \end{cases}$$

Show that $f_{xy}(0,0) \neq \mathrm{f}_{yx}(0,0)$. Also show that $f(x, \mathrm{y})$ does not satisfy Schwarz's theorem. **(6)**

3. (a) Prove that the function $f(z) = \dfrac{x^3(1+i) - y^3(1-i)}{x^2+y^2}(z \neq 0), f(0) = 0$ is continuous and that Cauchy-Riemann equations are satisfied at the origin, yet $f'(z)$ does not exist there. **(8)**

(b) Prove that if f has an isolated singularity at a then the point z = a is a removable singularity if and only if $\lim_{z \to a}(z-a)f(z) = 0$. **(8)**

(c) If an analytic function $f(z)$ has a pole of order m at $z = a$. Show that $\dfrac{1}{f(z)}$ has a zero of order of m at $z = a$. **(4)**

4. (a) Solve the following programming problem by graphical method. **(8)**

$Max\ z = 5x_1 + 7x_2$

subject to the constraints

$x_1 + x_2 \le 4$

$3x_1 + 8x_2 \le 24$

$10x_1 + 7x_2 \le 35$

$and\ x_1, x_2 \ge 0$

(b) Find the dual of the following L.P.P. **(4)**

$Min\ z = x_1 + x_2 + x_3$

subject to the constraints

$x_1 - 3x_2 + 4x_3 = 5$

$x_1 - 2x_2 \le 3$

$2x_2 - x_3 \ge 4$

$and\ x_1, x_2 \ge 0, x_3$ is *unrestricted in sign*

(c) Determine the optimum basic feasible solution to the following transportation problem. **(8)**

	D_1	D_2	D_3	D_4	Capacity
Q_1	1	2	3	4	6
Q_2	4	3	2	0	8
Q_3	0	2	2	1	10
Demand	4	6	8	6	24 (Total)

where Q_i and D_j denote the ith origin and jth destination, respectively.

PART - B

5. (a) Form a partial differential equation by eliminating the arbitrary function from

$$z = y^2 + 2f\left(\frac{1}{x} + \log y\right)$$ **(6)**

(b) Reduce the equation $\frac{\partial^2 z}{\partial x^2} = x^2\left(\frac{\partial^2 z}{\partial y^2}\right)$ to a canonical form. **(8)**

(c) Solve $\left(2D^2 - 5DD' + 2D'^2\right)z = 24(y - x)$. **(6)**

6. (a) Compute the positive root, correct to 3-significant figures by the method of bisection, of the equation $x^4 + x^2 - 1 = 0$. **(6)**

(b) Find the value of the integral $\int_0^1 \frac{dx}{1+x^2}$ by using Simpson's $\frac{1}{3}$ rule. Hence, obtain the approximate value of p. **(6)**

(c) Use Euler's method with $h = 0.2$ to find the solution of the differential equation

$$\frac{dy}{dx} = x + y, \mathrm{y}(0) = 0 \text{ in the range } 0 \le x \le 1.0$$ **(8)**

7. (a) Using the laws of Boolean Algebra, prove the following: **(4+4=8)**

(i) $(x + y)\cdot(\mathrm{x} + \mathrm{y}')\cdot(x' + z) = x \cdot z$

(ii) $\mathrm{z}\cdot(x + y) + x' \cdot z + y.z' = y + z$

(b) Convert the following numbers into the decimal number system: **(3+3=6)**

(i) $(10110101)_2$

(ii) $(A1E2)_{16}$

(iii) $(127.35)_8$

(c) Draw a circuit to realise the two input OR function and the two input AND function, using: **(3+3=6)**

(i) only the NOR gate

(ii) only the NAND gates

8. (a) Prove that the Moment of Inertia of a uniform right circular solid cone of mass *M*, height *h* and base radius *r*, about a diameter of its base is $\frac{M}{20}\left(3r^2 + 2h^2\right)$. **(7)**

(b) A rod of length *2a*, is suspended by a string of length *l*, attached to one end; if the string and rod revolve about the vertical with uniform angular velocity and their inclination to the vertical be θ and ϕ respectively, show that $\frac{3l}{a} = \frac{(4\tan\theta - 3\tan\phi)\sin\phi}{(\tan\phi - \tan\theta)\sin\theta}$ **(6)**

(c) Find the equation of motion of a compound pendulum using Hamilton's equation. **(7)**

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